EMIII (CBSGS) Paper / Subject Code: 49301 / APPLIED MATHEMATICS-III / NOV.2019
CMPN

Time Duration: 3Hr   Total Marks: 80

N.B.:1) Question no.1 is compulsory.
2) Attempt any three questions from Q.2to Q.6.
3) Figures to the right indicate full marks.

Q1. a) Find the Laplace transform of $e^{-4t} t \sin 3t$. [5]
b) Find the half-range cosine series for $f(x) = x$, $0 < x < 2$. [5]
c) Find $\nabla.\left(r\, \nabla \frac{1}{r^3}\right)$. [5]
d) Show that the function $f(z) = \sin z$ is analytic and find $f'(z)$ in terms of z. [5]

Q2. a) Find the inverse Z-transform of $F(z) = \frac{1}{(z-5)^3}$, $|z| < 5$. [6]
b) Find the analytic function whose imaginary part is $e^{-x}(y \sin y + x \cos y)$. [6]
c) Obtain Fourier series for the function $f(x) = x + x^2$, $-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$. [8]
Hence deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ and $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

Q3. a) Find $L^{-1}\left[\frac{1}{(s-a)(s-b)}\right]$ using convolution theorem. [6]
b) Is S= $\left\{\sin\left(\frac{\pi x}{4}\right), \sin\left(\frac{3\pi x}{4}\right), \sin\left(\frac{5\pi x}{4}\right), \ldots\ldots\right\}$ orthogonal in $(0, 1)$? [6]
c) Using Green's theorem in the plane evaluate $\int_c (xy + y^2)dx + (x^2)dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. [8]

Q4. a) Find Laplace transform of $f(t) = \begin{cases} \sin 2t & , o < t \le \frac{\pi}{2} \\ 0 & , \frac{\pi}{2} < t < \pi \end{cases}$ and $f(t) = f(t + \pi)$. [6]
b) Prove that a vector field $\bar{f}$ is irrotational and hence find its scalar potential $\bar{f} = (x^2 + xy^2)\, i + (y^2 + x^2 y) j$. [6]
c) Find the Fourier expansion for $f(x) = \sqrt{1 - \cos x}$ in $(0\ , 2\pi)$. Hence deduce that $\frac{1}{2} = \sum_1^{\infty} \frac{1}{4n^2 - 1}$. [8]

Q5.a) Use Gauss's Divergence Theorem to show that $\iint_S \nabla r^2 \overline{ds} = 6V$ where S is any closed surface enclosing a volume V. [6]
b) Find the Z-transform of $f(k) = b^k$, k< 0 . [6]
c) i) Find $L^{-1}\left[\frac{s}{(s-2)^6}\right]$. [8]
ii) Find $L^{-1}\left[\log\left(1 + \frac{a^2}{s^2}\right)\right]$.

Q6.a) Solve using Laplace transform [6]
$(D^2 + 9)y = 18t$, given that $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = 0$
b) Find the bilinear transformation which maps the points Z=∞, i, 0 onto W=0, i, ∞ . [6]
c) Find Fourier integral representation of $f(x) = e^{-|x|}$ $-\infty < x < \infty$. [8]

INFT/CBSGS/III **Paper / Subject Code: 49801 / APPLIED MATHEMATICS- III** / NOV. 2019

Time Duration: 3Hr **Total Marks: 80**

N.B.:1) Question no.1 is compulsory.
2) Attempt any three questions from Q.2to Q.6.
3) Figures to the right indicate full marks.

Q1. a) Find the Laplace transform of $e^{-4t}t\sin 3t$. **[5]**
b) Find the half-range cosine series for $f(x) = x\,,\ 0 < x < 2$. **[5]**
c) Find $\nabla.\left(r\,\nabla\frac{1}{r^3}\right)$. **[5]**
d) Show that the function $f(z) = \sin z$ is analytic and find $f'(z)$ in terms of z. **[5]**

Q2. a) Find the inverse Z-transform of $F(z) = \frac{1}{(z-5)^3}\ ,|z| < 5$. **[6]**
b) Find the analytic function whose imaginary part is $e^{-x}(y\sin y + x\cos y)$. **[6]**
c) Obtain Fourier series for the function $f(x) = x + x^2\ ,-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$. **[8]**
Hence deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ and $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

Q3. a) Find $L^{-1}\left[\frac{1}{(s-a)(s-b)}\right]$ using convolution theorem. **[6]**
b) Is S= $\left\{\sin\left(\frac{\pi x}{4}\right), \sin\left(\frac{3\pi x}{4}\right), \sin\left(\frac{5\pi x}{4}\right), \dots\dots\right\}$ orthogonal in (0 , 1)? **[6]**
c) Using Green's theorem in the plane evaluate $\int_c (xy + y^2)dx + (x^2)dy$ where C is the closed curve of the region bounded by $y = x$ *and* $y = x^2$. **[8]**

Q4. a) Find Laplace transform of $f(t) = \begin{cases} \sin 2t & ,o < t \le \frac{\pi}{2} \\ 0 & ,\frac{\pi}{2} < t < \pi \end{cases}$ and **[6]**
$f(t) = f(t + \pi)$.
b) Prove that a vector field $\bar{f}$ is irrotational and hence find its scalar potential **[6]**
$\bar{f} = (x^2 + xy^2)\,i + (y^2 + x^2y)j$.
c) Find the Fourier expansion for $f(x) = \sqrt{1 - \cos x}$ in $(0\ ,\ 2\pi)$. Hence deduce that $\frac{1}{2} = \sum_1^\infty \frac{1}{4n^2-1}$. **[8]**

Q5.a) Use Gauss's Divergence Theorem to show that $\iint_S \nabla r^2\overline{ds} = 6V$ where S is any closed surface enclosing a volume V. **[6]**
b) Find the Z-transform of $f(k) = b^k$, k< 0 . **[6]**
c) i) Find $L^{-1}\left[\frac{s}{(s-2)^6}\right]$. **[8]**
ii) Find $L^{-1}\left[\log\left(1 + \frac{a^2}{s^2}\right)\right]$.

Q6.a) Solve using Laplace transform **[6]**
$(D^2 + 9)y = 18t$, given that $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = 0$
b) Find the bilinear transformation which maps the points Z=∞, i, 0 onto W=0, i, ∞ . **[6]**
c) Find Fourier integral representation of $f(x) = e^{-|x|}\ \ -\infty < x < \infty$. **[8]**

E4166FBCBC26C4F0D54936B96AEE357A