

(3 Hours)

| Total marks : 80

- Note :-
 1) Question number 1 is compulsory.
 2) Attempt any three questions from the remaining five questions.
 3) Figures to the right indicate full marks.

- Q.1 a) Find the Laplace transform of $\sinh^5 t$. 05
 b) Find an analytic function whose imaginary part is $e^{-x}(y \cos y - x \sin y)$. 05
 c) Find the Fourier series for $f(x) = 1 - x^2$ in $(-1, 1)$. 05
 d) Evaluate $\int_C \bar{F} \cdot d\bar{r}$ where $\bar{F} = 2x i + (xz - y) j + 2z k$ from $O(0, 0, 0)$ to $P(3, 1, 2)$ along the line OP . 05
- Q.2 a) Find a cosine series of period 2π to represent $\sin x$ in $0 \leq x \leq \pi$. 06
 b) Find a, b, c if $\bar{F} = (axy + bz^3) i + (3x^2 - cz) j + (3xz^2 - y) k$ is irrotational. 06
 c) Find the image of the circle $|z| = k$ where k is real under the bilinear transformation $w = \frac{5-4z}{4z-3}$. 08
- Q.3 a) Prove that $J_{\frac{1}{2}}(x) = \tan x \cdot J_{-\frac{1}{2}}(x)$. 06
 b) Find the inverse Laplace transform of the following function by convolution theorem

$$\frac{(s+2)^2}{(s^2+4s+8)^2}$$
. 06
 c) Obtain the complex form of Fourier series for $f(x) = e^{ax}$ in $(-l, l)$ where a is not an integer. 08
- Q.4 a) Find the angle between the normals to the surface $xy = z^2$ at the points $(1, 4, 2)$ and $(-3, -3, 3)$. 06
 b) Prove that

$$x^2 J_n''(x) = (n^2 - n - x^2) J_n(x) + x J_{n+1}(x);$$

 $n = 0, 1, 2, \dots$. 06

- c)
 (i) Find the Laplace transform of $\sin \hat{a}t \sin at$. 04
 (ii) Find the Laplace transform of $te^{-4t} \sin 3t$. 04
- Q. 5 a) Prove that $J_2(x) = J''_0(x) - \frac{J_0'(x)}{x}$. 06
 b) If $v = e^x \sin y$, show that v is harmonic and find the corresponding analytic function. 06
 c) Find the Fourier series for $f(x)$ in $(0, 2\pi)$. 08

$$f(x) = \begin{cases} x, & 0 < x \leq \pi \\ 2\pi - x, & \pi \leq x < 2\pi \end{cases}$$

 Hence, deduce that

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$$
- Q. 6 a) Show that the set of functions $\cos nx$, $n = 1, 2, 3, \dots$ is orthogonal on $(0, 2\pi)$. 06
 b) Using Green's theorem evaluate $\int_C \bar{F} \cdot d\bar{r}$ where C is the curve enclosing the region bounded by $y^2 = 4ax$, $x = a$ in the plane $z = 0$ and

$$\bar{F} = (2x^2y + 3z^2)i + (x^2 + 4yz)j + (2y^2 + 6xz)k.$$
 06
 c) Use Laplace transform to solve 08

$$\frac{d^2y}{dt^2} + 4 \frac{dy}{dt} + 8y = 1 \text{ with } y(0) = 0, y'(0) = 1.$$
